

Application of Range Technique to Optimize Cost-Efficient Transportation Problem with Pentagonal Fuzzy Number

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ABSTRACT

In this paper, we focus on finding an optimal approximate solution for a specific type of optimization problem known as the fuzzy transportation problem, using pentagonal fuzzy number. In this context, the cost, supply and demand values associated with fuzzy transportation problems are represented as pentagonal fuzzy numbers. We convert these fuzzy numbers into crisp values using the range technique and solve the problem using the max-min method, which is applicable to transportation issues. The feasibility of our proposed method is demonstrated through numerical examples and the results are compared with those obtained from existing methods. Since our approach directly extends the classical method, it is straightforward to understand and can be easily applied by decision-makers to real-life transportation problems.

KEYWORDS

Optimization; Cost-Efficient; Transportation Problem; Pentagonal Fuzzy Number; Range-max-min Method.

1. Introduction

In today's highly competitive market, the pressure on organizations to find better ways to create and deliver value to customers becomes stronger. How and when to send the products to the customers in the quantities they want in a cost-effective manner becomes more challenging. Transportation models provide a powerful framework to meet this challenge. They ensure the efficient movement and timely availability of raw materials and finished goods.

Optimization is a central concept in operations research and it involves finding the best possible solution to a problem that satisfies given constraints. Optimization techniques are used in a wide range of fields, including engineering, finance, transportation, logistics and manufacturing Vikas S.Jadhav et al. (2023), [1]. Transportation problems are special kinds of problems in optimization. They are associated with real-world

activities that are managed with logistics. The problem for transportation includes transportation with a single manufacturing product in different supplies to a number of different destinations. The aim is to achieve the minimum total transportation cost of items that will satisfy the demands at various destinations. Furthermore, a problem of fuzzy transportation is one in which the transportation quantities regarding cost, supply and demand are all fuzzy. The objective of the present work is to calculate the shipped schedule for fuzzy transportation problem in order to minimize the total cost of fuzzy transportation while maintaining the limit of fuzzy supply and demand Rasha Jalal Mitilif et al. (2020),[2].

The basic transportation problem was originally developed by Hitchcock [3], Chanas [4] suggested an improved technique for the approximation solution of the fuzzy optimizing problem. In real-life problems, Zadeh (1965), [5], [6] had introduced the uncertainty theory, which is very useful for copying a large amount of data. There are so many instants; we need to find the maximum or minimum optimum solution for real-time problems. So, we need to achieve this efficient solution with the minimum impact of the environment and communities. Fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

In general, transportation problems are solved with the assumption that the transportation costs, supplies at sources and demands at destinations are specified in a precise way, i.e., in a crisp environment. However, in many cases, the decision maker may not be able to get precise values for the decision parameters for the transportation problem. If the nature of the information is vague, that is, if it has some lack of precision, then the corresponding coefficients or elements defining the problem can be formulated by means of fuzzy numbers and thus fuzzy transportation problems arise. In conventional transportation problems, it is assumed that the decision maker is sure about the precise values of transportation cost, availability and demand of the product. In real-world applications, all the parameters of the transportation problems may not be known precisely due to uncontrollable factors. This type of imprecise data is not always well represented by a random variable selected from a probability distribution. A fuzzy number may represent this data. So, a fuzzy decision-making method is needed here.

Aurora Nur Aini et al. (2021), [7] discussed the technique to explain the problem of transportation without finding an initial feasible solution with the help of the Zero Suffix method M.R. Fegade et al. (2012), [18] and the ASM method. Many authors, [8-10] defined and solved the fuzzy transportation problem with frequently applied fuzzy numbers namely triangular fuzzy numbers or trapezoidal fuzzy numbers. On the other hand, real-world problems usually involve more than four variables. To resolve those problems, the pentagonal fuzzy number is used.

Charles Rabinson et al. (2019), [11] proposed the allocation table process (ATM) to determine an initial basic feasible solution for fuzzy transportation problems with pentagonal fuzzy numbers. Apurba Panda et al. (2015), [12] discussed the special type of pentagonal fuzzy matrices and their algebraic natures. Also, they discussed the properties such as comparable, nilpotent and constant pentagonal fuzzy matrices. Pathinathan et al. [13-16] proposed four kinds of similarity measures for pentagonal fuzzy numbers and analyzed the similarity measure between two of them. Sankar Prasad Mondal et. al. (2017), [17] defined different forms of pentagonal fuzzy numbers and also addressed their arithmetic operations. The characteristics of pentagonal fuzzy numbers were extended to interval-valued fuzzy numbers and pentagonal fuzzy numbers in the literature.

In this paper, we proposed a new approach, called the recommended range method,

to solve the fuzzy transportation problem. It gives the minimum value for comparing all other existing methods, such as the Northwest Corner Method, VAM Method, LCM Method, and Zero Suffix Method. Finally, an illustrative example is given for the best understanding of the given algorithm.

The remainder of this paper is organized as follows. In section two, some basic definitions and arithmetic operations are reviewed. In section three, we attempt to introduce a mathematical formulation of the fuzzy transportation problem with pentagonal fuzzy numbers. In section four, deals proposed methodology, section five, numerical example is presented to illustrate the proposed method. Finally, conclusions and future work are presented in section six.

2. Preliminaries of Fuzzy Sets

This section provides fundamental definitions of fuzzy numbers, fuzzy sets, pentagonal fuzzy numbers, and some fuzzy arithmetic operations are discussed.

(a) Fuzzy Number: A is a fuzzy set on the real line \mathcal{R} , must satisfy the following conditions [5,6].

- i) $\mu_A(x_0)$ is piecewise continuous.
- ii) There exists at least one $x_0 \in \mathcal{R}$ such that $\mu_A(x_0) = 1$.
- iii) A must be regular and convex.

(b) Fuzzy Set: A fuzzy set A on the real line \mathcal{R} is defined as a set of ordered pairs

$$A = \{(x_0, \mu_A(x_0)) \mid x_0 \in \mathcal{A}, \mu_A(x_0) \in [0, 1]\}$$

where $\mu_A(x_0)$ is called the membership function of the fuzzy set A .

(c) Pentagonal Fuzzy Number: A fuzzy number A on \mathcal{R} is said to be a *pentagonal fuzzy number* (PFN), or a linear fuzzy number, denoted by:

$$A = (a_1, a_2, a_3, a_4, a_5),$$

if its membership function $\mu_A(x)$ has satisfies the following characteristics [11,12].

$$\mu_A(x) = \begin{cases} 0, & \text{if } x < a_1, \\ u_1 \frac{x - a_2}{a_3 - a_2}, & \text{if } a_1 \leq x \leq a_2, \\ 1, & \text{if } x = a_3, \\ 1 - (1 - u_2) \left(\frac{a_4 - x}{a_4 - a_3} \right), & \text{if } a_3 \leq x \leq a_4, \\ u_2 \left(\frac{a_5 - x}{a_5 - a_4} \right), & \text{if } a_4 \leq x \leq a_5, \\ 0, & \text{if } x > a_5. \end{cases} \quad (1)$$

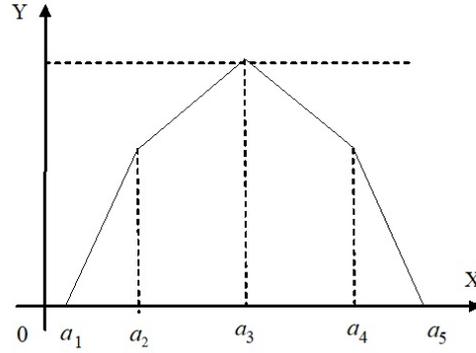


Figure 1.: Pentagonal Fuzzy Number

(d) Arithmetic Operations: Let $\bar{A} = (a_1, b_1, c_1, d_1, e_1)$ and $\bar{B} = (a_2, b_2, c_2, d_2, e_2)$ be two fuzzy numbers, where

$$a_1 \leq b_1 \leq c_1 \leq d_1 \leq e_1 \quad \text{and} \quad a_2 \leq b_2 \leq c_2 \leq d_2 \leq e_2.$$

then, the arithmetic operations are defined as [10].

i) **Addition:**

$$\bar{A} + \bar{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2, e_1 + e_2)$$

ii) **Subtraction:**

$$\bar{A} - \bar{B} = (a_1 - e_2, b_1 - d_2, c_1 - c_2, d_1 - b_2, e_1 - a_2)$$

iii) **Multiplication:**

$$\bar{A} * \bar{B} = \left(\frac{a_1}{5} \mu_\theta, \frac{b_1}{5} \mu_\theta, \frac{c_1}{5} \mu_\theta, \frac{d_1}{5} \mu_\theta, \frac{e_1}{5} \mu_\theta \right)$$

where $\mu_\theta = (a_2 + b_2 + c_2 + d_2 + e_2)$

iv) **Division:**

$$\bar{A} \div \bar{B} = \left(\frac{5a_1}{\mu_\theta}, \frac{5b_1}{\mu_\theta}, \frac{5c_1}{\mu_\theta}, \frac{5d_1}{\mu_\theta}, \frac{5e_1}{\mu_\theta} \right), \quad \text{if } \mu_\theta \neq 0,$$

where $\mu_\theta = (a_2 + b_2 + c_2 + d_2 + e_2)$

v) **Scalar Multiplication:**

$$k\bar{A} = \begin{cases} (ka, kb, kc, kd, ke), & \text{if } k > 0, \\ (ke, kd, kc, kb, ka), & \text{if } k < 0. \end{cases}$$

3. Mathematical Formulation of Fuzzy Transportation Problem

Let us consider a fuzzy transportation problem with m sources and n destinations whose parameters are represented by pentagonal fuzzy numbers. Let a_i ($a_i \geq 0$) denote the fuzzy availability at source i , and b_j ($b_j \geq 0$) denote the fuzzy requirement at destination j . Let c_{ij} be the fuzzy unit transportation cost from source i to destination j . Let x_{ij} denote the number of fuzzy units to be transported from source i to destination j .

The objective of the problem is to determine a feasible transportation plan that satisfies the demand at each destination using the available supply at each source while minimizing the total fuzzy transportation cost.

The mathematical formulation of the fuzzy transportation problem, under the assumption that the total supply equals the total demand, is given by:

$$\text{Minimize } z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij} \quad (2)$$

$$\text{subject to } \sum_{j=1}^n x_{ij} = a_i, \quad i = 1, 2, \dots, m, \quad (3)$$

$$\sum_{i=1}^m x_{ij} = b_j, \quad j = 1, 2, \dots, n, \quad (4)$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j, \quad (5)$$

$$x_{ij} \geq 0, \quad \forall i, j. \quad (6)$$

The general fuzzy transportation problem is explicitly represented as given below:

| Sources | Destinations | | | Supply |
|---------------|--------------|----------|----------|----------|
| | 1 | ... | n | |
| 1 | c_{11} | ... | c_{1n} | a_1 |
| \vdots | \vdots | \ddots | \vdots | \vdots |
| m | c_{m1} | ... | c_{mn} | a_m |
| Demand | b_1 | ... | b_n | |

4. Proposed Methodology

4.1. Range Technique-MAX-MIN Method – Algorithm

The range is defined as the difference between the maximum value and the minimum value

$$\text{Range} = \text{Maximum amount} - \text{Minimum amount.}$$

The algorithm for finding an optimal solution to the fuzzy transportation problem is described as follows:

- Step 1:** Construct the transportation table and examine whether the total demand equals the total supply. If $\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$, then proceed to Step 2.
- Step 2:** Using the range technique, convert the fuzzy transportation costs into crisp values for the given transportation problem.
- Step 3:** Compute the row-wise difference between the maximum and minimum values of each row and divide it by the number of columns of the cost matrix.
- Step 4:** Compute the column-wise difference between the maximum and minimum values of each column and divide it by the number of rows of the cost matrix.
- Step 5:** Determine the maximum of the resultant values and identify the corresponding minimum transportation cost. Allocate to the corresponding cell of the transportation matrix. If more than one maximum resultant value exists, any one of them may be selected.
- Step 6:** Repeat steps 1 to 5 until all allocations are completed.

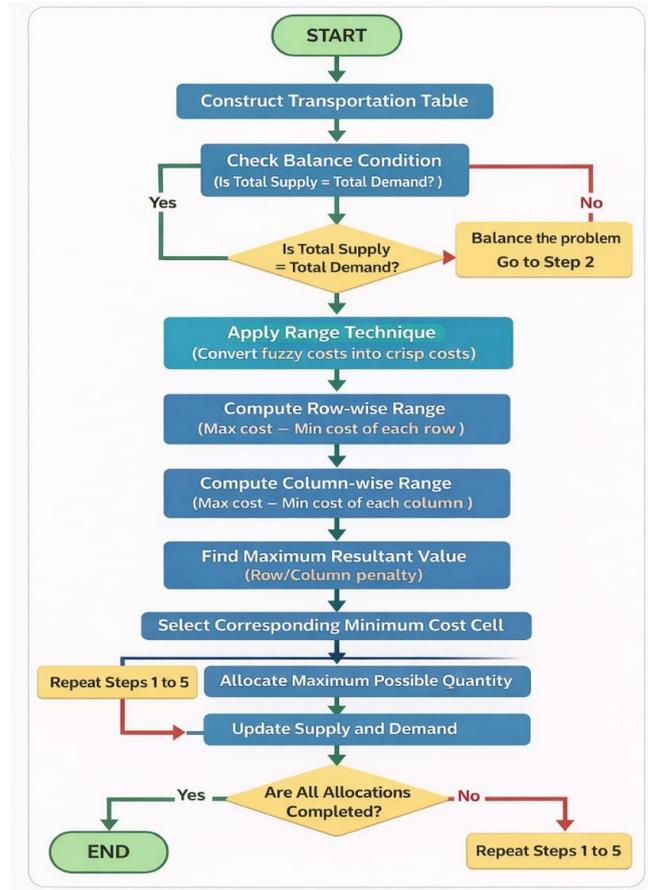


Figure 2.: Flow Chart of Proposed Algorithm

5. Numerical Example and Results

This section presents two numerical problems to demonstrate the methodology of the proposed algorithm. The first numerical problem considers a balanced fuzzy transportation problem, while the second numerical problem addresses an unbalanced fuzzy transportation problem.

5.1. Numerical Example - 1 Balanced Fuzzy Transportation Problem

Consider a balanced fuzzy transportation problem in which a product is manufactured by four factories, namely Factory 1, Factory 2, Factory 3 and Factory 4. The production capacities of the four factories are 30, 27, 40 and 50 units, respectively. The product is supplied to four stores, namely Store 1, Store 2, Store 3 and Store 4, with demand requirements of 20, 40, 34 and 53 units, respectively. The unit costs of fuzzy transportation are represented by fuzzy pentagonal numbers and are given below. The objective is to determine the fuzzy transportation plan such that the total production and transportation cost is minimized.

Table 1.: Given Dataset (FTP in Pentagonal Form)

| Sources | Store-1 | Store-2 | Store-3 | Store-4 | Supply |
|---------------|-----------------|------------------|------------------|------------------|--------|
| Factory-1 | (2, 4, 6, 8, 9) | (3, 5, 7, 8, 9) | (2, 4, 5, 6, 7) | (3, 4, 6, 7, 12) | 30 |
| Factory-2 | (0, 2, 5, 6, 8) | (4, 5, 6, 8, 11) | (2, 3, 5, 7, 11) | (1, 5, 6, 9, 11) | 27 |
| Factory-3 | (1, 2, 3, 4, 5) | (2, 3, 4, 6, 8) | (4, 5, 6, 8, 9) | (6, 7, 8, 9, 13) | 40 |
| Factory-4 | (3, 5, 6, 7, 8) | (1, 5, 6, 7, 8) | (2, 7, 8, 9, 10) | (3, 3, 4, 5, 9) | 50 |
| Demand | 20 | 40 | 34 | 53 | |

Solution By using the range, we have to convert fuzzy pentagonal numbers into a crisp value.

Table 2.: Defuzzied Transportation Problem

| | S_1 | S_2 | S_3 | S_4 | Capacity |
|--------|-----------|-----------|-----------|-----------|-----------|
| F_1 | 7 | 6 | 5 | 9 | 30 |
| F_2 | 8 | 7 | 9 | 10 | 27 |
| F_3 | 4 | 6 | 5 | 7 | 40 |
| F_4 | 5 | 7 | 8 | 6 | 50 |
| Demand | 20 | 40 | 34 | 53 | 147 |

Then find the maximum of the resultant values and find the corresponding minimum cost value and allocate the particular cost cell of the given matrix. If we have more than one maximum resultant benefit, we can select anyone.

Table 3.: First Iteration

| | S_1 | S_2 | S_3 | S_4 | Capacity | $\frac{\text{Max} - \text{Min}}{4}$ |
|-------------------------------------|-------------------|----------------------|-------------------|-------------------|----------|-------------------------------------|
| F_1 | 7 | 6 | 5^{30} | 9 | 30 | $\frac{4}{4} = 1$ |
| F_2 | 8 | 7 | 9 | 10 | 27 | $\frac{3}{4} = 0.75$ |
| F_3 | 4 | 6 | 5 | 7 | 40 | $\frac{3}{4} = 0.75$ |
| F_4 | 5 | 7 | 8 | 6 | 50 | $\frac{3}{4} = 0.75$ |
| Demand | 20 | 40 | 34 | 53 | | |
| $\frac{\text{Max} - \text{Min}}{4}$ | $\frac{4}{4} = 1$ | $\frac{1}{4} = 0.25$ | $\frac{4}{4} = 1$ | $\frac{4}{4} = 1$ | | |

Again, we find the maximum of the resultant values and find the corresponding minimum cost value and allocate the particular cost cell of the given matrix. If we have more than one maximum resultant values, we can select anyone.

Table 4.: Second Iteration

| | S_1 | S_2 | S_3 | S_4 | Capacity | $\frac{\text{Max} - \text{Min}}{4}$ |
|-------------------------------------|---------------------|----------------------|---------------------|---------------------|----------|-------------------------------------|
| F_1 | 7 | 6 | 5 | 9 | 30 | $\frac{4}{4} = 1$ |
| F_2 | 8 | 7 | 9 | 10 | 27 | $\frac{3}{4} = 0.75$ |
| F_3 | 4^{20} | 6 | 5 | 7 | 40 | $\frac{3}{4} = 0.75$ |
| F_4 | 5 | 7 | 8 | 6 | 50 | $\frac{3}{4} = 0.75$ |
| Demand | 20 | 40 | 34 | 53 | | |
| $\frac{\text{Max} - \text{Min}}{3}$ | $\frac{4}{3} = 1.3$ | $\frac{1}{3} = 0.03$ | $\frac{4}{3} = 1.3$ | $\frac{4}{3} = 1.3$ | | |

Table 5.: Final Iteration

| | S_1 | S_2 | S_3 | S_4 | Supply |
|---------------|-----------|-----------|-----------|-----------|-----------|
| F_1 | 7 | 6 | 5^{30} | 9 | 30 |
| F_2 | 8 | 7^{27} | 9 | 10 | 27 |
| F_3 | 4^{20} | 6^{13} | 5^4 | 7^3 | 40 |
| F_4 | 5 | 7 | 8 | 6^{50} | 50 |
| Demand | 20 | 40 | 34 | 53 | |

The total cost of transportation is given by

$$\text{Min } Z = 5 \times 30 + 4 \times 20 + 5 \times 4 + 7 \times 27 + 6 \times 13 + 7 \times 3 + 6 \times 50$$

Min $Z = 838^*$

Comparison with Existing Methods The comparison of the proposed method with the existing process is tabulated below, in which it is clearly shown that the proposed method provides the optimal results.

Table 6.: Comparative Results of the Existing Methods

| Methods | Optimal Solution |
|-------------------------|------------------|
| Northwest Corner Method | 898 |
| VA Method | 886 |
| Least Cost Method | 844 |
| Zero Suffix Method | 1,093 |
| Proposed Method | 838* |

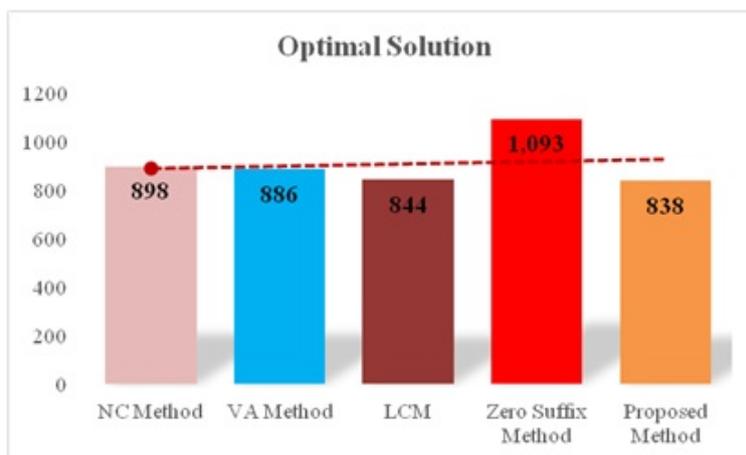


Figure 3.: Comparison of Optimization Results

5.2. Numerical Example–2 Unbalanced Fuzzy Transportation Problem

Consider the unbalanced fuzzy transportation problem. There are five factories F_1, F_2, F_3, F_4 , and F_5 , which supply goods to four dealers D_1, D_2, D_3 , and D_4 . The production capacities of these factories are 30, 27, 40, 50, and 24 units per month, respectively. The requirements from the dealers are 20, 40, 34, and 53 units per month, respectively. The following table gives the fuzzy transportation plan such that the total production and transportation cost is minimum.

Table 7.: Given Dataset (FTP in Pentagonal Form)

| | Dealer 1 | Dealer 2 | Dealer 3 | Dealer 4 | Capacity |
|-----------|-----------------|------------------|------------------|------------------|----------|
| Factory-1 | (2, 4, 6, 8, 9) | (3, 5, 7, 8, 9) | (2, 4, 5, 6, 7) | (3, 4, 6, 7, 12) | 30 |
| Factory-2 | (0, 2, 5, 6, 8) | (4, 5, 6, 8, 11) | (2, 3, 5, 7, 11) | (1, 5, 6, 9, 11) | 27 |
| Factory-3 | (1, 2, 3, 4, 5) | (2, 3, 4, 6, 8) | (4, 5, 6, 8, 9) | (6, 7, 8, 9, 13) | 40 |
| Factory-4 | (3, 5, 6, 7, 8) | (1, 5, 6, 7, 8) | (2, 7, 8, 9, 10) | (3, 3, 4, 5, 9) | 50 |
| Factory-5 | (1, 3, 4, 6, 7) | (0, 2, 4, 3, 5) | (1, 3, 4, 6, 9) | (2, 4, 7, 8, 12) | 24 |
| Demand | 20 | 40 | 34 | 53 | |

Solution:

By using the range technique, we have to convert fuzzy pentagonal numbers into a crisp value.

Table 8.: Defuzzified Transportation Problem

| | D_1 | D_2 | D_3 | D_4 | Capacity |
|--------|-------|-------|-------|-------|----------|
| F_1 | 7 | 6 | 5 | 9 | 30 |
| F_2 | 8 | 7 | 9 | 10 | 27 |
| F_3 | 4 | 6 | 5 | 7 | 40 |
| F_4 | 5 | 7 | 8 | 6 | 50 |
| F_5 | 6 | 5 | 8 | 10 | 24 |
| Demand | 20 | 40 | 34 | 53 | |

The given matrix is unbalanced, and we add a zero column to balance the given matrix.

Table 9.: First Iteration

| | D_1 | D_2 | D_3 | D_4 | D_5 | Capacity |
|--------|-------|-------|-------|-------|-------|----------|
| F_1 | 7 | 6 | 5 | 9 | 0 | 30 |
| F_2 | 8 | 7 | 9 | 10 | 0 | 27 |
| F_3 | 4 | 6 | 5 | 7 | 0 | 40 |
| F_4 | 5 | 7 | 8 | 6 | 0 | 50 |
| F_5 | 6 | 5 | 8 | 10 | 0 | 24 |
| Demand | 20 | 40 | 34 | 53 | 24 | |

Then find the maximum of the resultant values and find the corresponding minimum

cost value and allocate the particular cost cell of the given matrix. If we have more than one maximum resultant values, we can select anyone.

Table 10.: Second Iteration

| | D_1 | D_2 | D_3 | D_4 | D_5 | Capacity | $\frac{\text{Max} - \text{Min}}{5}$ |
|-------------------------------------|---------------------|---------------------|---------------------|---------------------|-----------------------|----------|-------------------------------------|
| F_1 | 7 | 6 | 5 | 9 | 0 | 30 | $\frac{9}{5} = 1.8$ |
| F_2 | 8 | 7 | 9 | 10 | 0²⁴ | 27 | $\frac{10}{5} = 2$ |
| F_3 | 4 | 6 | 5 | 7 | 0 | 40 | $\frac{7}{5} = 1.4$ |
| F_4 | 5 | 7 | 8 | 6 | 0 | 50 | $\frac{8}{5} = 1.6$ |
| F_5 | 6 | 5 | 8 | 10 | 0 | 24 | $\frac{10}{5} = 2$ |
| Demand | 20 | 40 | 34 | 53 | 24 | | |
| $\frac{\text{Max} - \text{Min}}{5}$ | $\frac{4}{5} = 0.8$ | $\frac{2}{5} = 0.4$ | $\frac{4}{5} = 0.8$ | $\frac{4}{5} = 0.8$ | $\frac{0}{5} = 0$ | | |

Again, we find the maximum of the resultant values and find the corresponding minimum cost value and allocate the particular cost cell of the given matrix. If we have more than one maximum resultant values, we can select anyone.

Table 11.: Third Iteration

| | D_1 | D_2 | D_3 | D_4 | Capacity | $\frac{\text{Max} - \text{Min}}{4}$ |
|-------------------------------------|---------------------|-----------------------|---------------------|---------------------|----------|-------------------------------------|
| F_1 | 7 | 6 | 5 | 9 | 30 | $\frac{4}{4} = 1$ |
| F_2 | 8 | 7 | 9 | 10 | 27 | $\frac{3}{4} = 0.75$ |
| F_3 | 4 | 6 | 5 | 7 | 40 | $\frac{3}{4} = 0.75$ |
| F_4 | 5 | 7 | 8 | 6 | 50 | $\frac{3}{4} = 0.75$ |
| F_5 | 6 | 5²⁴ | 8 | 10 | 24 | $\frac{5}{4} = 1.25$ |
| Demand | 20 | 40 | 34 | 53 | | |
| $\frac{\text{Max} - \text{Min}}{5}$ | $\frac{4}{5} = 0.8$ | $\frac{2}{5} = 0.4$ | $\frac{4}{5} = 0.8$ | $\frac{4}{5} = 0.8$ | | |

The same procedure will be followed again and again until we reach the final allocation.

Table 12.: Final Iteration

| | D_1 | D_2 | D_3 | D_4 | D_5 | Capacity |
|---------------|-----------|-----------|-----------|-----------|-----------|-----------|
| F_1 | 7 | 6 | 5^{30} | 9 | 0 | 30 |
| F_2 | 8 | 7^3 | 9 | 10 | 0^{24} | 27 |
| F_3 | 4^{20} | 6^{13} | 5^4 | 7^3 | 0 | 40 |
| F_4 | 5 | 7 | 8 | 6^{50} | 0 | 50 |
| F_5 | 6 | 5^{24} | 8 | 10 | 0 | 24 |
| Demand | 20 | 40 | 34 | 53 | 24 | |

The following total transportation cost is obtained:

$$\begin{aligned} \text{Min } Z &= 0 \times 24 + 5 \times 24 + 5 \times 30 + 4 \times 20 \\ &\quad + 5 \times 4 + 7 \times 3 + 6 \times 13 + 7 \times 3 + 6 \times 50 \end{aligned}$$

$$\text{Min } Z = 790^*$$

Comparison with Existing Methods

The comparison of the proposed method with the existing plan is tabulated below, in which it is clearly shown that the proposed method provides the optimal results.

Table 13.: Comparative Results of the Existing Methods

| Methods | Optimal Solution |
|-------------------------|------------------|
| Zero Suffix Method | 1022 |
| Northwest Corner Method | 898 |
| VA Method | 838 |
| Least Cost Method | 844 |
| Proposed Method | 790* |

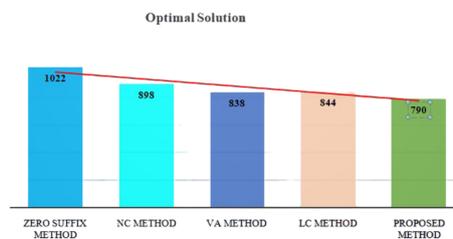


Figure 4.: Comparison of Optimization Results

6. Conclusion and Future Work

In this paper, we have presented a range-based approach to solve transportation problems modeled with pentagonal fuzzy numbers. The fuzzy cost coefficients were converted into crisp values using a simple ranking technique and solved via the max–min method. Numerical results for both balanced and unbalanced scenarios demonstrate that our proposed method achieves lower transportation costs compared to traditional and existing fuzzy methods. Moreover, this approach is easy to implement, computationally efficient and applicable in real-world transportation decision-making under uncertainty.

The proposed range-based approach could be expanded to address multi-objective transportation problems, time-dependent cost structures, and real-time logistics systems that involve dynamic fuzzy parameters. Additionally, future research may investigate the hybridization of this method with metaheuristic or machine learning techniques to improve the robustness and scalability of solutions for large-scale transportation networks.

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Conflict of Interest

Authors do not have any conflict of interest.